An Adaptive Order Execution Strategy for VWAP Tracking

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***Abstract*—This paper presents an adaptive execution strategy for high frequency trading in the Stock Exchange of Thailand (SET). The objective of the strategy is to buy or sell stocks for a specific amount in SET to match the actual daily market VWAP (Volume-Weighted Average Price) as much as possible in a specified time interval for day trading. By sending required amounts of stocks to acquire or liquidate in a specific time interval, our adaptive execution algorithm will calculate a volume profile and a ratio between Limit Order (LO) and Market Order (MO) and send the order at each time step until the end of the time interval. The algorithm was tested in an order simulation system by using a historical data set of SET.**

***Index Terms*—high frequency trading, limit Order, market order, stock, order execution, volume profile**

I. Introduction

In general, selling or buying stock is more or less has some effects on the market price. The market price impact can be separated into 2 types: permanent price impact and temporary price impact. Typically, an execution algorithm is used to generate child orders in order to reduce the market impact. Some important factors to reduce the price impact are a volume size and an execution order time [1]. If the volume size is large compared to the market volume, the market price might be increased or decreased depending on customer buying or selling volume demands. Normally, the order is not immediately matched. The price that a broker aims to get when sending the order is called the public stock price, while the trading price might be different. If the price is changed in between the trading period, the broker has to absorb this changed value. Therefore, managing child orders in a time horizon is important in order to minimize the cost of trading.

Recently, the mean-variance optimal adaptive execution [2] was proposed as the variation of the static Almgren-Chriss (AC) Framework [3] [4] for selling stock units. The random process used is the key difference from the static AC method. It can be said that the dynamic strategy is adjusted according to the market price.

Up to now, there are various researches about managing child orders. One of well-known algorithms for managing a large order into child orders is Time-Weight Average Price (TWAP), which divides an order into a fixed volume by a number of trading times.

Not only volume management is important, but also a type of execution orders. There are two types of execution order, which are limit order (LO) and market order (MO). A MO is immediately executed from the best bid or best offer price, while LO stores an order in a matching queue for an expected price. The benefit of MO is that an order can be executed immediately, while LO has a possibility not to be executed. However, LO has more possibility to get a better price than MO. Thus, balancing the amount of LO and MO is an important trading parameter.

In this paper, an adaptive execution strategy is proposed for high frequency trading in the Stock Exchange of Thailand (SET). The objective of the strategy is to buy or sell stocks for a specific amount in SET to match the actual daily market VWAP (Volume-Weighted Average Price) as much as possible in a specified time interval for day trading. By sending required amounts of stocks to acquire or liquidate in a specific time interval, our adaptive execution algorithm will calculate a volume profile and a ratio between Limit Order (LO) and Market Order (MO) and send the order at each time step until the end of the time interval. The algorithm was tested in an order simulation system by using a historical data set of SET.

The remaining of the paper is organized as follows. Section 2 gives a brief introduction of the proposed method: the adaptive order execution strategy. Section 3 presents how the performance of the adaptive order execution strategy is evaluated. Section 4 describes both how data from SET are used and experiments results are explained. The discussion is also presented in this section. The conclusion of the work and remarks are presented in Section 5.

II. Adaptive Order Execution Strategy

A. Volume Profile

A volume profile is a predicted buying or selling volume, which is used for planning for an execution period of interest. In this paper, the data in the same day is used for the volume profile prediction.

The following notations of trading times are used: let $t_i, i = 1, 2, \ldots, N$ be the order time step, $t_N = NT$ be the last order time, N is the number of sending orders, and T is the period of sending order time.

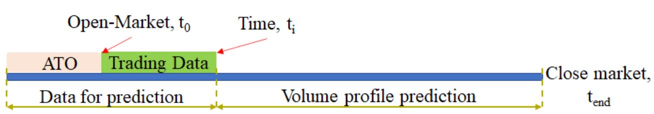

Fig. 1. Volume profile prediction

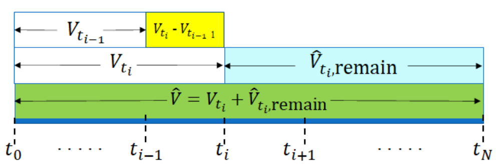

Fig. 2. Volume profile parameter definition

Let $\hat{V}_{t_i,\text{remain}}$ be the predicted trading volume from the current time t_i to the market close time t_{end} as shown in Fig. 1 and Fig. 2. This volume is predicted by using the executed trading volume in ATO period and the trading volume from the opening market time t_0 to the current time t_i. There are two steps for planning a volume profile. First, the predicted volume is estimated by using linear regression:

$$\hat{V}_{t_i,\text{remain}} = \beta_1 V_{\text{ATO}} + \beta_2 V_{t_i} + \epsilon, \tag{1}$$

where $\hat{V}_{t_i,\text{remain}}$ is the predicted volume from the current time t_i to the end of trading time t_{end}. V_{ATO} is the trading volume in ATO period, and V_{t_i} is the trading volume from the opening market time t_0 to the current time t_i. β_1 and β_2 are the parameter coefficients and ϵ is the model error.

After the predicted volume $\hat{V}_{t_i,\text{remain}}$ is obtained, the total predicted market volume $\hat{V}$ from t_0 to t_{end} can be calculated as

$$\hat{V} = V_{t_i} + \hat{V}_{t_i,\text{remain}}. \tag{2}$$

Note that $\hat{V}$, V_{t_i}, and $\hat{V}_{t_i,\text{remain}}$ do not include the trading volume in ATO period V_{ATO} and the trading volume in ATC period V_{ATC} .

The next step is to plan an executed volume v_i by using the following formula:

$$v_i = \min\left\{ \max\left\{ \left[\frac{V_{t_i} - V_{t_{i-1}}}{\hat{V} - V_{t_{i-1}}} \right] Q_i, 0 \right\}, Q_i \right\} \tag{3}$$

$$Q_i = C - \sum_{i=1}^{i-1} v_i, \tag{4}$$

where v_i is the executed volume of the order at time t_i, V_{t_i} is the trading volume from the opening market time t_0 to the current time t_i, $\hat{V}$ is the predicted volume from the market opening time t_0 to the end of trading time t_{end}, Q_i is the remaining volume that have to send an order, and C is the total volume to be bought or sold defined as an input of execution order. In this work, $T = 1$ minute and $N = 260$ are used.

B. MO and LO Ratios

There are two kinds of execution orders: market order (MO) and limit order (LO). The parameter MO ratio $r_{i,\text{MO}}$ is the ratio of market order to limit order for each time t_i. In each order time, the MO ratio is adjusted. The value of these ratios is not equal for buy order and sell order. The MO ratio of buy order can be calculated by using the following equation:

$$r_{i,\text{MO, buy}} = \frac{vol_{\text{price>bestBid}}}{vol_{\text{total}}} \tag{5}$$

$$r_{i,\text{LO, buy}} = 1 - r_{i,\text{MO, buy}} \ , \tag{6}$$

where $r_{i,\text{MO, buy}}$ and $r_{i,\text{LO, buy}}$ are the market and the limit order buy ratios at time t_i, respectively, $vol_{\text{price>bestBid}}$ is trading volume that has the price more than the best bid price, vol_{bestBid} is trading volume at the best bid price, $vol_{\text{price<bestBid}}$ is trading volume at the price less than the best bid price and vol_{total} is the total trading volume at all calculated price. The data of trading volumes are obtained at time t_i. Note that, the maximum value of $r_{i,\text{MO}}$ and $r_{i,\text{LO}}$ are equal to 1.

In order to clarify the calculation of $r_{i,\text{MO}}$ and $r_{i,\text{LO}}$, a calculation example is shown as follows. Assume that there are 1,500 stock units at price 7 THB, 1,000 stock units at best bid price 8 THB, and 2,000 stock units at price 9 THB for buy trading order. The system keeps the previous time data, which is the change of best bid price from 7 THB to 8 THB. The calculation of $r_{i,\text{MO, buy}} = vol_{\text{price9}}/vol_{\text{total}} = 0.44$ and $r_{i,\text{LO, buy}} = 1 - 0.44 = 0.56$.

For sell order, the $r_{i,\text{MO, sell}}$ can be calculated by using $r_{i,\text{LO, buy}}$ formula in (7) and (8).

$$r_{i,\text{LO, sell}} = \frac{vol_{\text{price>bestBid}}}{vol_{\text{total}}} \tag{7}$$

$$r_{i,\text{MO, sell}} = 1 - r_{i,\text{LO, sell}} \tag{8}$$

From above example, the $r_{i,\text{MO, sell}} = r_{i,\text{LO, buy}} = 0.56$ and $r_{i,\text{LO, sell}} = r_{i,\text{MO, buy}} = 0.44$.

C. Adaptive Order Execution Strategy

Fig. 3 shows the overall block diagram of the proposed method. The algorithm aims to mimic the market behavior. The input data is the data of the calculation day.

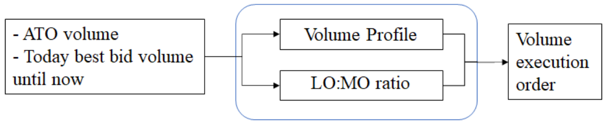

Fig. 3. The Proposed Method: Adaptive Order Execution Strategy

This algorithm is the combination of an adjustment of volume profile and the MO ratio, with respect to the ATO price and trading data before the calculation point period. The algorithm is predicted the whole day volume and divided the whole day volume into child orders. The parameter in the MO ratio manages the amount of LO and MO orders for each time t_i.

III. PERFORMANCE EVALUATION

A. TWAP

The Time Weight Average Price (TWAP) is one of the methods that traders use for reducing a market impact. The volume and time of orders are fixed by using the average value. Let the total trading time be T_{trade}. The total trading time is divided by N trading times. Let $\Delta T = T_{\text{trade}}/N$ be the time spacing. Hence, the TWAP equation can be written as

$$\text{TWAP} = \frac{\sum_{i=1}^{N} \Delta T\, p_{t_i}}{T_{\text{trade}}} = \frac{\sum_{i=1}^{N} p_{t_i}}{N}, \tag{9}$$

where p_{t_i} is the average market price at trading time t_i.

B. VWAP

The Volume Weight Average Price (VWAP) is one of trading benchmarks used by investors. The VWAP is calculated by

$$\text{VWAP} = \frac{\sum_{i=1}^{N} m_{t_i}\, p_{t_i}}{V}, \tag{10}$$

where m_{t_i} is the market volume and $p_{t_i} > 0$ is the average market price at time t_i. Moreover, V is the total trading volume, in one day or in interested period, can be calculated by

$$V = \sum_{i=1}^{N} m_{t_i}. \tag{11}$$

C. Performance Evaluation

1) The Percentage Difference between Market VWAP and Proposed Method's VWAP: In this paper, performance evaluation is compared by using the percentage difference between the market VWAP and the proposed method's VWAP, which can be calculated by the following equation:

$$\%\Delta_{\text{VWAP}} = \frac{\text{VWAP}_{\text{Sim}} - \text{VWAP}_{\text{Market}}}{\text{VWAP}_{\text{Market}}} \times 100\%, \tag{12}$$

where $\%\Delta_{\text{VWAP}}$ is the difference between VWAP of the market and VWAP from the simulation of the proposed method, $\text{VWAP}_{\text{Market}}$ is VWAP of the market and VWAP_{Sim} is VWAP from the simulation. Note that for a buy order, the fewer $\%\Delta_{\text{VWAP}}$ means the lesser cost. On the other hand, for a sell order, the more $\%\Delta_{\text{VWAP}}$ means the more profit.

2) Percentage of Remaining LO: When sending an LO order, the order is in the queue waiting for a matched order. When the market is almost at the close time, there is a possibility that LO order is not matched. In this case, $\%\text{Vol}_{\text{remain}}$ is considered at the stop execution time, which can be calculated by the following equation:

$$\%\text{Vol}_{\text{remain}} = \frac{vol_{\text{LO, remain}}}{vol_{\text{total}}} \times 100\%, \tag{13}$$

where $vol_{\text{LO, remain}}$ is the remaining LO order that is not success for a buying or a selling order.

IV. EXPERIMENTS AND RESULTS

In this section, the proposed method is evaluated by using the historical real data from SET: Stock Exchange of Thailand. The VWAP is selected to measure the performance of the proposed method. This section is separated into 4 parts. Data information from the Stock Exchange of Thailand is presented in the first part. In the second part, the results between the proposed method and TWAP are compared. The experiment in changing the input volume from 1 million THB to 10 million THB is explained in the third part. Some volume adjustment following the price direction prediction is presented in the last part.

A. Data form SET: Stock Exchange of Thailand

The simulation is based on real historical stock data from SET. About 28 tickers are randomly chosen in total as representatives of tested results as shown in TABLE I. In TABLE I, the average standard deviation (S.D.) of daily intra return of best bid price is the criteria for standard deviation (S.D.). The result in this work is an average of 100 test days.

TABLE I
STOCK SAMPLES

Group	S.D.	Ticker
High S.D.	0.009705 - 0.013750	DELTA, ERW, SPVI, PLANB, KCE, PSL, SMT, BWG, DNA
Mid S.D.	0.006863 - 0.007159	IRPC, MC, TTA, EPG, CPN, RS, SAT, GPSC, EA, CKP
Low S.D.	0.002503 - 0.004517	JASIF, TRUEIF, TTW, CPALL, ADVANC, TPPIPP, SCC, TIP, SCCC

Note that if the data of a ticker is not enough, that ticker is not calculated.

B. Comparison with TWAP

In this section, the main purpose is to compare the result between TWAP and the proposed method by using VWAP and the percentage of the remaining LO order as performance measurements. The MO ratios of TWAP in this experiment are 70, 50 and 30, while the MO ratio of the proposed method is calculated by using the equation (5) and (6). The minimum execution order is 100 units for each child order.

Fig. 4 illustrates the example of a predicted volume profile and an MO ratio in one day comparing to the market volume profile. In the case, the predicted market volume is similar to the real market volume, i.e., $V_{t_i} - V_{t_{i-1}}$ is similar to $\hat{V} - V_{t_{i-1}}$.

TABLE II
AVERAGE PERCENTAGE DIFFERENCE BETWEEN MARKET VWAP AND OTHERS OF BUY ORDER

Group	$\%\Delta_{\text{VWAP, Sim}}$	$\%\Delta_{\text{VWAP, MO30}}$	$\%\Delta_{\text{VWAP, MO50}}$	$\%\Delta_{\text{VWAP, MO70}}$
High S.D.	0.1294	0.1450	0.2127	0.2827
Mid S.D.	0.0874	0.1155	0.1600	0.2089
Low S.D.	0.1596	0.1625	0.1966	0.2098

Fig.5 shows the result of percentage difference between market VWAP and proposed method on CPALL for 100 testing days.

In the case that the total predicted volume is more than the real market volume, the volume of the order tends to increase a lot more than the beginning of the day in order to make no ordered volume left at the end of the day. On the other hand, if the predicted volume is less than the real market volume, it tends to complete the task faster than the plan.

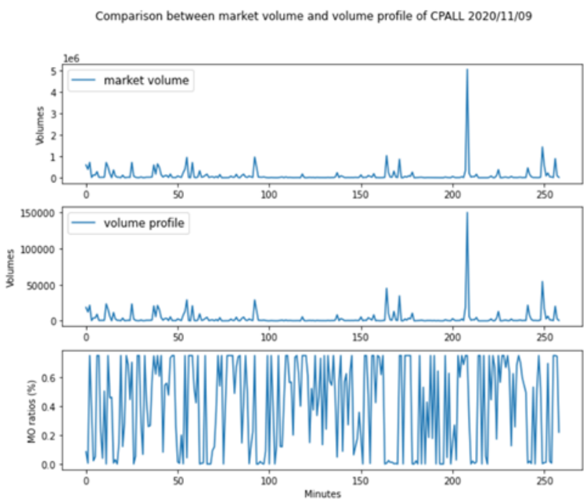

Fig. 4. buy order example in one day of CPALL (Top) market volume (Mid) predicted volume profile (bottom) MO ratio.

The result shows that the average percentage difference of VWAP of the proposed method is outstanding in all variations both buy and sell orders. An average buy result of high S.D., mid S.D. and low S.D. of the proposed method are 0.1294, 0.0874 and 0.1596 respectively, while -0.3669, -0.2222 and -0.1692 are the result of sell order in sequence.

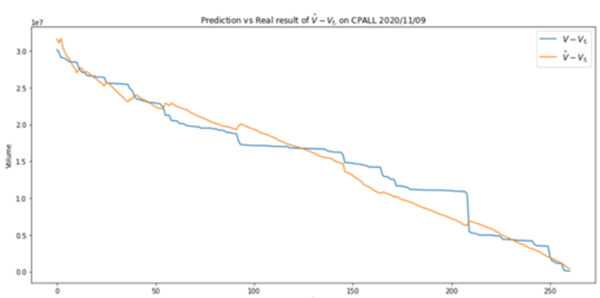

Fig. 5. example of predicted market volume is similar to real market volume

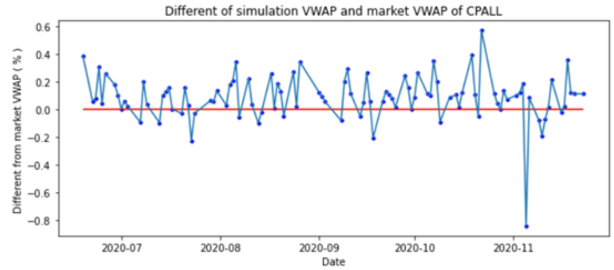

Fig. 6. Percentage difference of market VWAP and proposed on CPALL of 100 testing days

TABLE III
AVERAGE PERCENTAGE DIFFERENCE BETWEEN MARKET VWAP AND OTHERS OF SELL ORDER

Group	$\%\Delta_{\text{VWAP, Sim}}$	$\%\Delta_{\text{VWAP, MO30}}$	$\%\Delta_{\text{VWAP, MO50}}$	$\%\Delta_{\text{VWAP, MO70}}$
High S.D.	-0.3669	-0.4745	-0.5021	-0.5284
Mid S.D.	-0.2222	-0.2466	-0.2777	-0.3089
Low S.D.	-0.1692	-0.1839	-0.2019	-0.2206

C. Volume Variation Management

In this experiment, input volume varies from 1 million THB to 10 million THB. It is assumed that the volume of the order is small compared to the market volume, so there is no price impact in the simulation.

The result shows that there are no outstanding differences when varies the order value either buy and sell.

The limitation of this algorithm is that there is no market impact effecting in this simulation, so the result of varying the order value might not be in the same way as the real trading.

D. Volume Adjustment

In this experiment, both volume and MO ratio in each time t_i is considered to adjust following the price direction prediction. The price direction prediction is predicted, by SVM classifier with linear kernel, separating into 3 classes: increasing, unchanging and decreasing. The calculation data is 500 data points, before the calculation point, in total to predict the price direction in the next 5 minutes as in fig.7. After getting the price direction class, the volume will be adjusted as follows. If the price tends to rise, increase the buy volume and MO ratio for a buy order, while decrease both sell volume and MO ratio for sell order. On the other hand, if the price tends to fall off, decrease both volume and MO ratio for a buy order, but increase both volume and MO ratio for a sell order.

The percentage of VWAP difference between $\text{VWAP}_{\text{Market}}$ and VWAP of the interested method is used to determine the result of this topic. There are 5 variation methods of

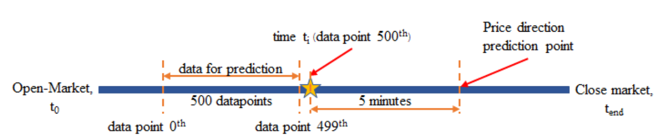

Fig. 7. Price direction prediction data

TABLE IV
AVERAGE PERCENTAGE DIFFERENCE BETWEEN MARKET VWAP AND PROPOSED METHOD OF 1M TO 10M THB BUY ORDER

Buy	$\%\Delta_{\text{VWAP, Sim}}$									
Group	1M	2M	3M	4M	5M	6M	7M	8M	9M	10M
Hight S.D.	0.1212	0.1232	0.1255	0.1279	0.1305	0.1339	0.1361	0.1389	0.1414	0.1438
Mid S.D.	0.1033	0.0993	0.0990	0.0998	0.1010	0.1025	0.1042	0.1056	0.1070	0.1086
Low S.D.	0.1642	0.1622	0.1618	0.1618	0.1618	0.1623	0.1631	0.1638	0.1644	0.1647

TABLE V
AVERAGE PERCENTAGE DIFFERENCE BETWEEN THE MARKET VWAP AND THE PROPOSED METHOD'S VWAP OF 1M TO 10M THB SELL ORDER

Sell	$\%\Delta_{\text{VWAP, Sim}}$									
Group	1M	2M	3M	4M	5M	6M	7M	8M	9M	10M
Hight S.D.	-0.3694	-0.3690	-0.3698	-0.3713	-0.3722	-0.3737	-0.3754	-0.3772	-0.3789	-0.3807
Mid S.D.	-0.2387	-0.2316	-0.2299	-0.2296	-0.2298	-0.2307	-0.2316	-0.2328	-.02337	-0.2348
Low S.D.	-0.2018	-0.1908	-0.1860	-0.1836	-0.1827	-0.1820	-0.1815	-0.1809	-0.1809	-0.1812

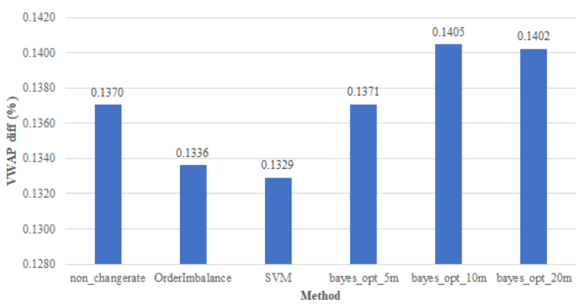

Fig. 8. comparison of $\%\Delta_{\text{VWAP}}$ of buy order for volume adjustment

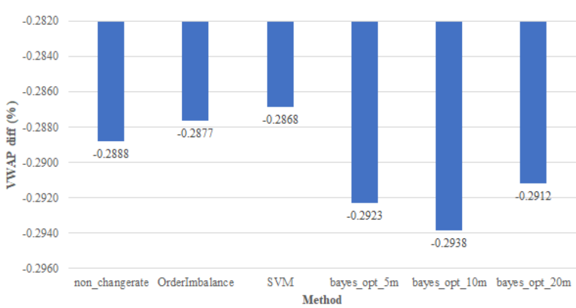

Fig. 9. comparison of $\%\Delta_{\text{VWAP}}$ of sell order for volume adjustment

the volume adjustments, which are order imbalance, SVM, Bayesian optimization [5], [6] with 5 minutes, 10 minutes, and 20 minutes window, compared to no adjustment. The change rate is used for adjusting both parameters. The maximum number of change rates is increased or decrease $\pm 100\%$ from the current volume profile and MO ratio at time t_i, but not more than the total execution order. The Bayesian optimization will be adjusted both volume profile and MO ratio automatically.

In both Fig. 8 and Fig.9, SVM gives the best performance following by order imbalance and no volume adjustment respectively. The reason why Bayesian optimization is not outstanding over non-adjustment of change rate might be because of the system error.

V. CONCLUSION

The adaptive order execution for VWAP tracking is proposed in order to manage to sell or buy the order. The algorithm predicts the whole day market volume, and divides the order into child orders as a volume profile. Moreover, the type of order, LO and MO, are also considered as MO ratio. The algorithm is adjusted the ratio between LO and MO by considering the ratio of the predicted market volume and best bid volume. The purpose of the algorithm is to mimic the market behavior. The simulation is tested based on the real data from SET as an average of 100 testing days, and uses about 29 tickers as a representative result.

There are two experiments for not considering the price change. The first experiment is comparing with TWAP method. The result shows that the proposed method is outstanding in all groups if considering the percentage difference of VWAP. In another experiment, varying the order value from 1 million to 10 million THB, there is no different in each variation. The average of percentage difference of VWAP is less than 0.2% in all group for buy order and less than 0.4% for sell order.

In order to adjust the order, price prediction direction is concerned to adjust volume profile and MO ratio at time t_i. The result of SVM in volume adjustment is outstanding over other methods, which are order imbalance, non-adjustment and Bayesian optimization respectively.

In the future, the MO ratio will be adjusted following the price prediction direction in order to make more potential in price change.

ACKNOWLEDGMENT

This research was supported by Capital Market Development Fund (CMDF) (Contract No: CMDF_CONT0005_2563).

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